I. Zeeman Effect - Revisited

- No external $\vec{B}$-field ($\vec{B}_{ext} = 0$), we have $\vec{B}_{int}$ that leads to spin-orbit interaction (thus $\hat{H}'_{so}$) [always there!]
- Zeeman Effect: There is an externally applied $\vec{B}$-field ($\vec{B}_{ext} \neq 0$)

  Additional interaction energies (terms) in Hamiltonian

  - $\vec{L} \rightarrow \vec{\mu}_L = \frac{-e}{2m_e}\vec{L}$ leads to $(-\vec{\mu}_L \cdot \vec{B}_{ext})$ ← considered in "Normal Zeeman effect"
  - $\vec{S} \rightarrow \vec{\mu}_S = \frac{-e}{m_e}\vec{S}$ leads to $(-\vec{\mu}_S \cdot \vec{B}_{ext})$

  ↑ new terms in Hamiltonian when $\vec{B}_{ext} \neq 0$

Cannot avoid it as $\vec{S}$ is already invoked in $\hat{H}_{so}$ (spin-orbit interaction)

$$\hat{H}'_{Zeeman} = \hat{H}'_z = -\hat{\vec{\mu}}_L \cdot \vec{B}_{ext} - \hat{\vec{\mu}}_S \cdot \vec{B}_{ext} = -\hat{\vec{\mu}}_{total} \cdot \vec{B}_{ext}$$

$$= \frac{e}{2m_e}\hat{\vec{L}} \cdot \vec{B}_{ext} + \frac{e}{m_e}\hat{\vec{S}} \cdot \vec{B}_{ext} \qquad \left[\vec{\mu}_{total} = \vec{\mu}_L + \vec{\mu}_S\right]$$

$$= \frac{e}{2m_e}\left(\hat{\vec{L}} + 2\hat{\vec{S}}\right) \cdot \vec{B}_{ext} \quad (30)$$

"Zeeman term" that goes into Hamiltonian

[Note: $\vec{L} + 2\vec{S} \neq \vec{J}$ as $\vec{J} = \vec{L} + \vec{S}$]

- Atom in $\vec{B}_{ext}$: The full QM problem is to solve TISE with

$$\boxed{\hat{H} = \hat{H}_{atom} + \underbrace{\hat{H}'_{so}}_{\text{spin-orbit interaction } f(r)\,\vec{S}\cdot\vec{L}\ [\text{independent of } \vec{B}_{ext}]} + \underbrace{\hat{H}'_z}_{\text{Zeeman } \frac{e}{2m_e}(\vec{L}+2\vec{S})\cdot\vec{B}_{ext}} \left(+ \hat{H}'_{rel}\right)} \quad (31)$$

$\hat{H}'_{rel}$: even a relativistic correction [ignore here!] [less important in atoms other than hydrogen]

What to do?

Think like a physicist!

$$\hat{H} = \hat{H}_{atom} + \underbrace{\hat{H}'_{so} + \hat{H}'_z}} \quad (32)$$

- Two things to do? Do the more important thing (part) first and do it more seriously (rigorously)!

- Which term is more important?
  - No standard answer!
  - Depends on situation, i.e. $\vec{B}_{ext}$

The Big Picture

- Strong $\vec{B}_{ext}$ field: $\hat{H} = \underbrace{\left(\hat{H}_{atom} + \hat{H}'_z\right)}_{\text{handle more accurately}} + \underbrace{\hat{H}'_{so}}_{\text{perturbation}}$ (32a)

  ($\hat{H}_z \propto \vec{B}_{ext}$) more important

- Weak $\vec{B}_{ext}$ field: $\hat{H} = \underbrace{\left(\hat{H}_{atom} + \hat{H}'_{so}\right)}_{\text{treated under spin-orbit interaction}} + \underbrace{\hat{H}'_z}$ (32b)

  B_{int} more important (order ~ 1 Tesla)

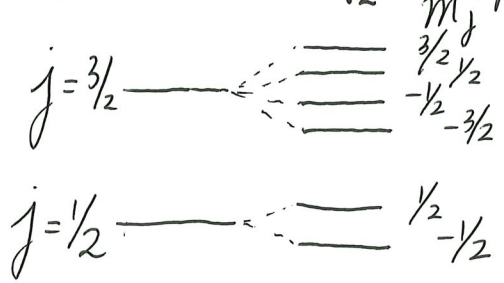
  $\hat{H}'_{so}$

Strong $\vec{B}_{ext}$

- Treat $\hat{H}'_z = \frac{eB_{ext}}{2m}(\hat{L}_z + 2\hat{S}_z)$ first
  - states labelled by $(n, \ell, m_\ell, s, m_s)$ are eigenstates of $(\hat{H}_{atom} + \hat{H}'_z)$
  - $\hat{H}'_z$ removes degeneracies behind m_ℓ and m_s
- With $\psi_{n\ell m_\ell s m_s}$, treat $\hat{H}'_{so}$ as perturbation

Weak $\vec{B}_{ext}$

- Treat $\hat{H}'_{so} = f(r)\,\vec{S}\cdot\vec{L}$ first
  - states labelled by (n, ℓ, s, j, m_j) are good for $\hat{H}'_{so}$
- Then treat $\hat{H}'_z$ as perturbation



[due to $\hat{H}'_{so}$] (fine structure) ↑ [due to $\hat{H}'_z$]

[It is $|\vec{B}_{ext}|$ ($\hat{H}'_z$) vs $|\vec{B}_{int}|$ ($\hat{H}'_{so}$) that decides which term is more important]

($\vec{B}_{int} \sim 1$ Tesla)

Keep this Big Picture in mind as we fill in the math.

Moral of the Story

- Work on the more important thing first! (Common Sense!)
  (人生道理)

Q: What if $\hat{H}'_{so}$ and $\hat{H}'_{z}$ are comparable in effects?

- Most difficult case
- tune B_{ext} to avoid this

(a) Strong Field Zeeman Effect

$$\hat{H}_{atom}\,\psi_{n\ell m_\ell s m_s} = E_{n\ell}\,\psi_{n\ell m_\ell s m_s}$$

($s = \frac{1}{2}$ always; $E_{n\ell}$: for $U(r)$ [spherically symmetric, assumed for atoms]; [0^{th} order problem])

→ for H-atom, becomes $E_n = -\frac{13.6}{n^2}$ eV ("n only" for H-atom)

- The same $\{\psi_{n\ell m_\ell(s) m_s}\}$ also solve $(\hat{H}_{atom} + \hat{H}'_z)$

$$\because\ \hat{H}'_z\,\psi_{n\ell m_\ell(s) m_s} = \frac{e}{2m} B_{ext}\left(\hat{L}_z + 2\hat{S}_z\right)\psi_{n\ell m_\ell(s) m_s} = \frac{e\hbar}{2m_e} B_{ext}\,(m_\ell + 2m_s)\,\psi_{n\ell m_\ell(s) m_s}$$

($\hat{L}_z$ picks up $(m_\ell\hbar)$; $\hat{S}_z$ picks up $(m_s\hbar)$; $(m_\ell + 2m_s)$: eigenvalue of $\hat{H}'_z$)

$$\therefore\ (\hat{H}_{atom} + \hat{H}'_z)\,\psi_{n\ell m_\ell(s) m_s} = \left[-\frac{13.6}{n^2} + \mu_B B_{ext}\,(m_\ell + 2m_s)\right]\psi_{n\ell m_\ell(s) m_s} \qquad (33)$$

Solved $(\hat{H}_{atom} + \hat{H}'_z)$ exactly!

$-\frac{13.6}{n^2}$: for H-atom (or $E_{n\ell}$) [removed degeneracies behind m_ℓ and m_s]

$\mu_B B_{ext}\,(m_\ell + 2m_s)$: eigenvalues now depend on m_ℓ, m_s

∴ For $(\hat{H}_{atom} + \hat{H}'_z)$, the description $\psi_{n\ell m_\ell(s) m_s}$ is preferred over $\psi_{n\ell(s) j m_j}$

Aside:

Recall $\psi_{n\ell m_\ell(s) m_s} = \underbrace{\psi_{n\ell m_\ell}(r,\theta,\phi)}_{R_{n\ell}(r)\cdot Y_{\ell m_\ell}(\theta,\phi)} \cdot \begin{cases} \alpha_z & (\text{if } m_s = +1/2) \text{ "up"} \\ \beta_z & (\text{if } m_s = -1/2) \text{ "down"} \end{cases}$

∴ $\hat{L}_z \psi_{n\ell m_\ell} = m_\ell \hbar \, \psi_{n\ell m_\ell}$ (∵ $Y_{\ell m_\ell}(\theta,\phi)$ is eigenstate of $\hat{L}_z$ with eigenvalue $m_\ell\hbar$)

Similarly, $\hat{S}_z \alpha_z = \underbrace{\frac{+\hbar}{2}}_{m_s\hbar\ (m_s=+1/2)} \alpha_z$; $\hat{S}_z \beta_z = \underbrace{-\frac{\hbar}{2}}_{m_s\hbar\ (m_s=-1/2)} \beta_z$ [∵ $\hat{S}_z$ sees α_z, β_z]

∴ Eq. (33) follows.

$$\hat{H} = \underbrace{(\hat{H}_{atom} + \hat{H}_z')}_{\text{treated exactly (plays the role of } \hat{H}_0\text{) using } \psi_{n\ell m_\ell(s) m_s}} + \hat{H}_{so}' \leftarrow \text{treat it perturbatively}$$

- Apply 1st order perturbation theory:

$$E_{so}^{(1)} = \int \psi^*_{n\ell m_\ell(s) m_s} \left(f(r)\, \underbrace{\hat{\vec{S}}\cdot\hat{\vec{L}}}_{\hat{S}_x\hat{L}_x + \hat{S}_y\hat{L}_y + \hat{S}_z\hat{L}_z}\right) \psi_{n\ell m_\ell(s) m_s}\, d\tau \qquad \text{[used 1st order formula]}$$

$$= \langle f(r)\rangle \left[\langle \hat{S}_x\rangle^{0^+}\langle \hat{L}_x\rangle^{0^+} + \langle \hat{S}_y\rangle^{0}\langle \hat{L}_y\rangle^{0} + \langle \hat{S}_z\rangle\langle \hat{L}_z\rangle \right]$$

$\langle f(r)\rangle \sim \int |R_{n\ell}(r)|^2 f(r)\, r^2 dr$ (depends on n, ℓ)

$\langle \hat{S}_x\rangle^{0^+}$: expectation value over spin state specified by m_s

$\langle \hat{L}_y\rangle^{0}$: expectation value over orbital AM state specified by ℓ, m_ℓ

$\langle \hat{S}_z\rangle \to m_s\hbar$, $\langle \hat{L}_z\rangle \to m_\ell\hbar$

$$= m_\ell m_s \hbar^2 \langle f(r)\rangle \qquad \text{(Key result)} \qquad (34)$$

+ See next page on handling the spin part, e.g. $\psi_{n\ell m_\ell(s)\,1/2} = \psi_{n\ell m_\ell}(r,\theta,\phi)\cdot\alpha_z$

Arguments

Q: Why $\langle \hat{S}_x \rangle = 0$? $\psi_{n\ell m_\ell (s) m_s}$ is a state with definite S_z $\begin{cases} +\frac{\hbar}{2} & (m_s = 1/2) \\ -\frac{\hbar}{2} & (m_s = -1/2) \end{cases}$

- Such state is NOT an eigenstate of $\hat{S}_x$ ($\because [\hat{S}_z, \hat{S}_x] \neq 0$)
- Go through the thought of multiple copies of states [definite $(s) m_s$] and measure S_x

$$\begin{cases} \rightarrow 50\% \quad S_x = +\frac{\hbar}{2} \\ \rightarrow 50\% \quad S_x = -\frac{\hbar}{2} \end{cases} > \langle S_x \rangle = 0$$

Q: Why $\langle \hat{L}_x \rangle = 0$ for $\psi_{n \underline{\ell m_\ell} (s) m_s}$?

↖ state of definite L_z (equals $m_\ell \hbar$)

Such state is NOT an eigenstate of $\hat{L}_x$

... [same thoughts] $\langle \hat{L}_x \rangle = 0$

Thus Eq.(34) follows.

- also works for $\langle \hat{S}_y \rangle = 0$ & $\langle \hat{L}_y \rangle = 0$

Math Details: Zooming into $\langle \hat{S}_x \rangle = 0$ ← Why?

$\psi_{n\ell m_\ell (s) m_s}$ — m_s related to spin; $|s\ m_s\rangle$

$$|s\ m_s\rangle = \begin{cases} |\tfrac{1}{2}\ \tfrac{1}{2}\rangle \text{ for } S_z = \frac{\hbar}{2} \text{ (up) state} & \text{also called } \alpha_z \\ |\tfrac{1}{2}\ -\tfrac{1}{2}\rangle \text{ for } S_z = \frac{-\hbar}{2} \text{ (down) state} & \text{also called } \beta_z \end{cases}$$

$\therefore \langle \hat{S}_x \rangle$ is either $\langle \alpha_z | \hat{S}_x | \alpha_z \rangle$ or $\langle \beta_z | \hat{S}_x | \beta_z \rangle$

- "States of definite S_z have completely uncertain S_x" ($\because [\hat{S}_z, \hat{S}_x] \neq 0$)
  $\therefore \langle \hat{S}_x \rangle = 0$ (by argument)
- By Math:
$$\langle \alpha_z | \hat{S}_x | \alpha_z \rangle = (1^* \ 0^*) \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 0$$
$$\langle \beta_z | \hat{S}_x | \beta_z \rangle = (0^* \ 1^*) \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0$$
- $\langle \hat{L}_x \rangle = \int Y^*_{\ell m_\ell}(\theta, \phi)\ \hat{L}_x\ Y_{\ell m_\ell}(\theta, \phi)\, d\Omega = 0$ (same argument)
- Similar consideration for $\langle \hat{S}_y \rangle$ and $\langle \hat{L}_y \rangle$

Putting results together (Strong-field):

$$E_{n\ell m_\ell(s) m_s} \approx \underbrace{-\frac{13.6}{n^2}}_{\text{ignored everything (QM I) (H-atom)}} + \underbrace{\mu_B B_{ext}(m_\ell + 2m_s)}_{\text{due to } \hat{H}'_z \text{ (thus } B_{ext}\text{) treated exactly}} + \underbrace{m_\ell m_s \hbar^2 \langle f(r) \rangle}_{\text{due to } \hat{H}'_{so} \text{ 1st order perturbation}} \quad (35)$$

(Done!)

Example: $\ell = 1$ states (p states) $\quad m_\ell = \begin{cases} 1 \\ 0 \\ -1 \end{cases}$ and $m_s = \begin{cases} +\frac{1}{2} \text{ (up)} \\ -\frac{1}{2} \text{ (down)} \end{cases}$ total of 6 states

- ignore everything (1st term in Eq. (35)): $-\frac{13.6}{n^2}$ ——————— $(\ell = 1)(6 \text{ states})$

6 states have

m_ℓ	m_s	$m_\ell + 2m_s$ *	$m_\ell m_s$ +
1	$+\frac{1}{2}$	2	$+\frac{1}{2}$
0	$+\frac{1}{2}$	1	0
1	$-\frac{1}{2}$	0	$-\frac{1}{2}$
−1	$+\frac{1}{2}$	0	$-\frac{1}{2}$
0	$-\frac{1}{2}$	−1	0
−1	$-\frac{1}{2}$	−2	$+\frac{1}{2}$

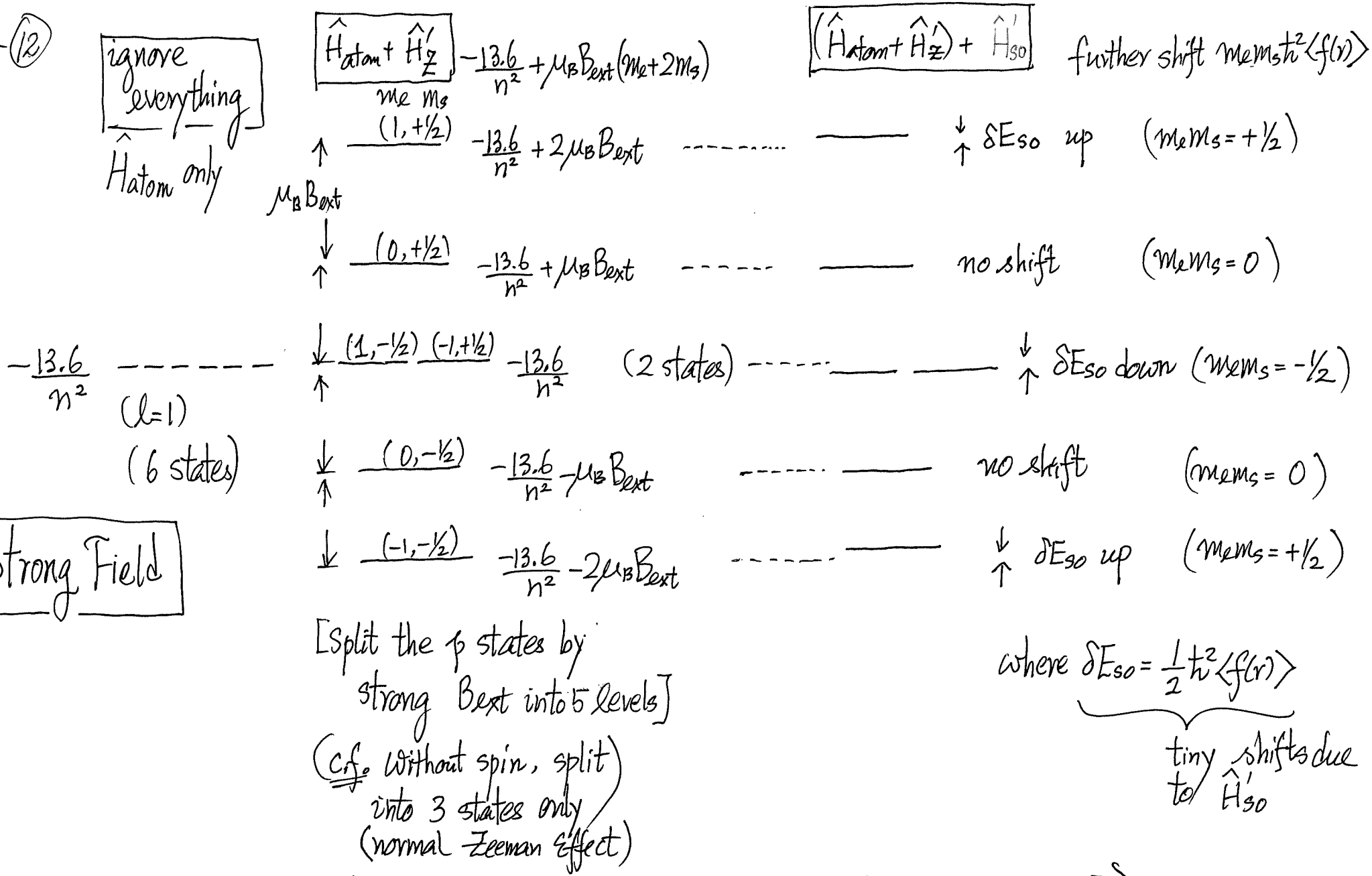
* Useful for $\hat{H}'_z$ term

\+ Useful for $\hat{H}'_{so}$ term (See Eq. (35))

ignore everything

$\hat{H}_{atom}$ only

Strong Field



[Split the p states by strong B_{ext} into 5 levels]

(c.f. without spin, split into 3 states only (normal Zeeman Effect))

where $\delta E_{SO} = \frac{1}{2}\hbar^2 \langle f(r) \rangle$ — tiny shifts due to $\hat{H}'_{SO}$

- In any case, the key message is that degenerate states are split in $\vec{B}_{ext}$.
- Similar consideration for other ℓ.
- Transitions [e.g. $2p \to 1s$] lead to Zeeman Effect (splitting of spectral line in $\vec{B}_{ext}$)

- In atomic physics, the strong field case is called the "Paschen-Back Effect"
- The observed phenomena are similar to Normal Zeeman Effect

(b) Weak Field Zeeman Effect

$\hat{H}_{atom}$ term: $\psi_{n\ell m_\ell(s) m_s}$ and $\psi_{n\ell(s) j m_j}$ are equally good (both with $E_{n\ell}$)

Weak field $\Rightarrow$ $\underbrace{\hat{H}'_{so}}_{B_{int}}$ is more important than $\underbrace{\hat{H}'_z}_{B_{ext}\ (weak)}$

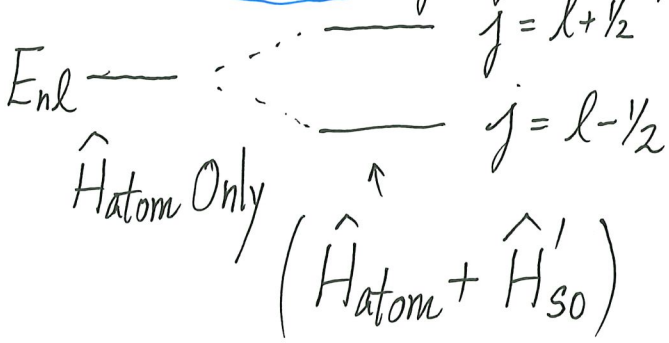
Do $(\hat{H}_{atom} + \hat{H}'_{so})$ first (Done! Fine structure)

$\psi_{n\ell(s) j m_j}$ are preferred over $\psi_{n\ell m_\ell(s) m_s}$ due to $\hat{H}'_{so}$

makes $\vec{J}$ important

not L_z, not S_z

they lost their identities



Then $\hat{H}_z' = -\hat{\vec{\mu}}_{total} \cdot \vec{B}_{ext} = \frac{e}{2m_e} \vec{B}_{ext} \cdot (\hat{\vec{L}} + 2\hat{\vec{S}})$ is treated as perturbation

$(\hat{H}_{atom} + \hat{H}_{so}')$ requires $\psi_{n\ell(s)jm_j}$ ← "unperturbed states"

$$E^{(1)}_{Zeeman} = \langle n\ell(s)jm_j | \left(-\hat{\vec{\mu}}_{total} \cdot \vec{B}_{ext}\right) | n\ell(s)jm_j \rangle \quad (1^{st} \text{ order perturbation})$$

$$= \langle n\ell(s)jm_j | \frac{e}{2m_e} \vec{B}_{ext} \cdot \underbrace{(\hat{\vec{L}} + 2\hat{\vec{S}})}_{\neq \hat{\vec{J}}} | n\ell(s)jm_j \rangle \quad (36)$$

Not easy! Need further approximation

Problem arises from:

$$\vec{\mu}_{total} = \frac{-e}{2m_e}(\vec{L} + 2\vec{S}) = \frac{-e}{2m}(\vec{J} + \overbrace{\vec{S}}^{extra!}) \neq g\left(\frac{-e}{2m_e}\right)\vec{J}$$

↑ Not true

as in $\begin{cases} \vec{\mu}_L = (1)\left(\frac{-e}{2m_e}\right)\vec{L} \\ \vec{\mu}_S = (2)\left(\frac{-e}{2m_e}\right)\vec{S} \end{cases}$

Idea: It will be nice if $\vec{\mu}_{total} \approx \vec{\mu}_{J\,(approx)} = \underbrace{g}_{\text{some number}} \left(\frac{-e}{2m_e}\right) \vec{J}$ (37)

If so, $-\vec{B}_{ext} \cdot \hat{\vec{\mu}}_{J\,(approx)} \left| n\,l\,(s)\,j\,m_j \right\rangle$

$= \frac{e}{2m_e} g\, \vec{B}_{ext} \cdot \hat{\vec{J}} \left| n\,l\,(s)\,j\,m_j \right\rangle$ $\quad (\vec{B}_{ext} = B_{ext}\hat{z})$

$= \frac{eB_{ext}}{2m_e} g\, \hat{J}_z \left| n\,l\,(s)\,j\,m_j \right\rangle$ $\quad (\hat{J}_z |j\,m_j\rangle = m_j \hbar |j\,m_j\rangle)$

$= B_{ext} \frac{e\hbar}{2m_e} g\, m_j \left| n\,l\,(s)\,j\,m_j \right\rangle$ $\quad$ (Done!? What's g?)

and

$$E^{(1)}_{Zeeman} \cong \left\langle n\,l\,(s)\,j\,m_j \right| \left(-\vec{B}_{ext} \cdot \hat{\vec{\mu}}_{J\,(approx)}\right) \left| n\,l\,(s)\,j\,m_j \right\rangle$$

$$= g\,\mu_B B_{ext} m_j \qquad (38)$$

($\psi_{n\,l\,(s)\,j\,m_j}$ are normalized)

∴ If approximation (37) is made, life becomes easy & $E^{(1)}_{Zeeman}$ is given by Eq. (38)

How to approximate $\vec{\mu}_{total} = \frac{-e}{2m_e}(\vec{L}+2\vec{S})$ reasonably by $\vec{\mu}_{J(approx)} = g\left(\frac{-e}{2m_e}\right)\vec{J}$?

Idea: Project $\underbrace{(\vec{L}+2\vec{S})}_{=\vec{J}+\vec{S}}$ along $\vec{J}$ (as the approximation)

along $\vec{J}$ ok

Project $\vec{S}$ in direction of $\vec{J}$ and ignore the rest

Approximate $\vec{S}$ by:

$$\left(\vec{S}\cdot\frac{\vec{J}}{|J|}\right)\left(\frac{\vec{J}}{|J|}\right) = \frac{(\vec{S}\cdot\vec{J})}{J^2}\vec{J} = \frac{J^2-L^2+S^2}{2J^2}\vec{J}$$

$\frac{\vec{J}}{|J|}$: unit vector in direction of $\vec{J}$

$\vec{J} = \vec{L}+\vec{S}$

$\Rightarrow \vec{L} = \vec{J}-\vec{S}$

$\Rightarrow L^2 = J^2+S^2-2\vec{S}\cdot\vec{J}$

$\Rightarrow \vec{S}\cdot\vec{J} = \frac{J^2-L^2+S^2}{2}$

$$\therefore\ \vec{J}+\vec{S} \approx \left(1+\frac{J^2-L^2+S^2}{2J^2}\right)\vec{J} \qquad \text{(now along } \vec{J}\text{)}$$

Then $E^{(1)}_{Zeeman}$ becomes (for $\vec{B}_{ext}=B_{ext}\hat{z}$):

$$E^{(1)}_{Zeeman} \approx \langle n\,\ell(s)\,j\,m_j \left| B_{ext}\frac{e\hbar}{2m_e} m_j \left(1+\frac{\hat{J}^2-\hat{L}^2+\hat{S}^2}{2\hat{J}^2}\right)\right| n\,\ell(s)\,j\,m_j\rangle$$

$$= \langle n\,\ell(s)\,j\,m_j \left| B_{ext}\mu_B m_j \left(1+\frac{j(j+1)-\ell(\ell+1)+s(s+1)}{2j(j+1)}\right)\right| n\,\ell(s)\,j\,m_j\rangle$$

$$= g_{Landé}\,\mu_B B_{ext}\,m_j \qquad (39) \qquad [\text{Done!}]$$

where

$$g_{Landé} = \underbrace{1+\frac{j(j+1)-\ell(\ell+1)+s(s+1)}{2j(j+1)}}$$

↯ Landé g-factor

a number (not necessarily integer) (order 1)

← depending on j and how that value of j comes about (ℓ and s) (How important is spin-orbit interaction)

- Same form as previously found in normal Zeeman effect

Normal [ignored spin]	Including spin (anomalous Zeeman effect)
$E_{Zeeman} = g_L \mu_B B m_\ell$ ($g_L = 1$)	$E_{Zeeman} \approx g_{Landé} \mu_B B m_j$

This is because we have effectively approximated $\vec{\mu}_{total}$ by

$$\vec{\mu}_{J(approx)} = g_{Landé}\left(\frac{-e}{2m}\right)\vec{J} \qquad (40)$$

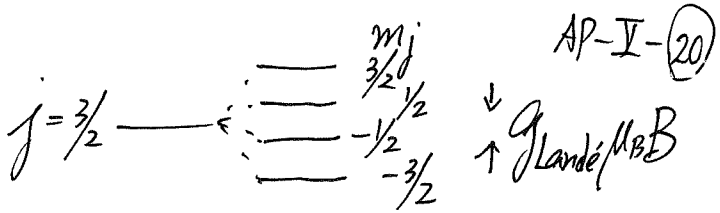
which has the same form† as $\vec{\mu}_L = g_L\left(\frac{-e}{2m_e}\right)\vec{L}$

$\vec{\mu}_S = g_S\left(\frac{-e}{2m_e}\right)\vec{S}$

---

† But $g_{Landé}$ takes on a value that depends on j AND (ℓ and s) [Term Symbol]

$$\therefore\ E^{(1)}_{Zeeman} = g_{Landé}\,\mu_B\,B\,m_j$$



removes degeneracy due to m_j

Example

$\ell = 1$ (p states)

(6 states with spin)

Ignoring everything	+	Including $\hat{H}'_{so}$ only	+	Weak field with $\hat{H}'_z$

$\hat{H}_{atom}$ only

Weak Field

n p states $-\frac{13.6eV}{n^2}$ ------ (6 states)

(e.g. 2p)

(H-atom)

$\ell = 1$

$(S = 1/2)$

(4 states) $j = 3/2$ — m_j: $+3/2$, $+1/2$, $-1/2$, $-3/2$

(2 states) $j = 1/2$ — m_j: $+1/2$, $-1/2$

With $\hat{H}'_{so}$ only, energy depends on n and j (see fine-structure discussion)

[no external field here, $B_{ext} = 0$]

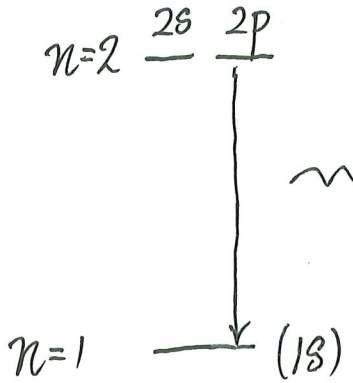
B_{ext} lifts degeneracy as

$E^{(1)}_{Zeeman} = g_{Landé}\mu_B m_j B_{ext}$

depending on m_j

Consequences : Look into the first Lyman line (Weak Field)

$\hat{H}_{atom}$ only



$\rightsquigarrow \hbar\omega = \Delta E = -\frac{13.6}{4} - \left(-\frac{13.6}{1}\right) \approx 10.2\,eV$

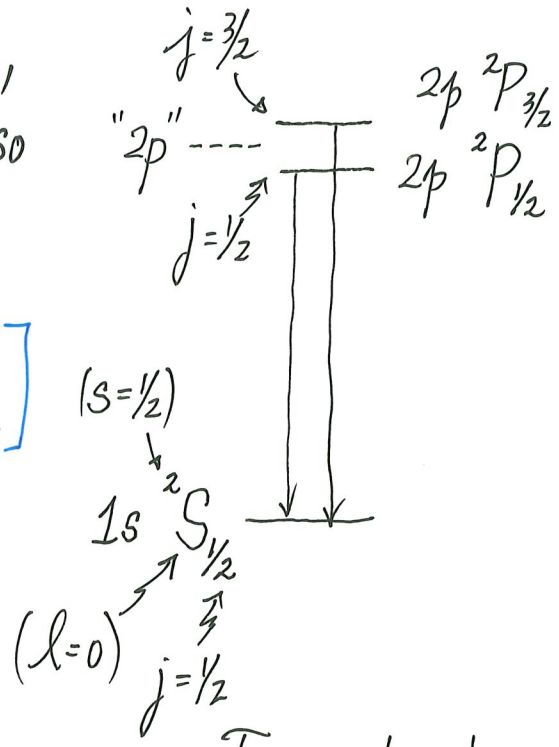
spectrum

one line at ~10.2eV (in Lyman Series)

$\hat{H}_{atom} + \hat{H}'_{so}$

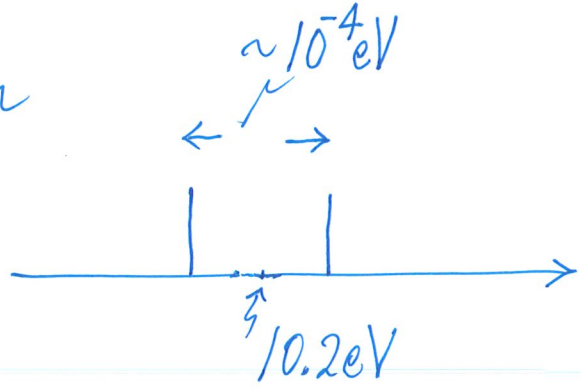
$(\vec{B}_{ext} = 0)$

[spin-orbit coupling only]



Fine structure

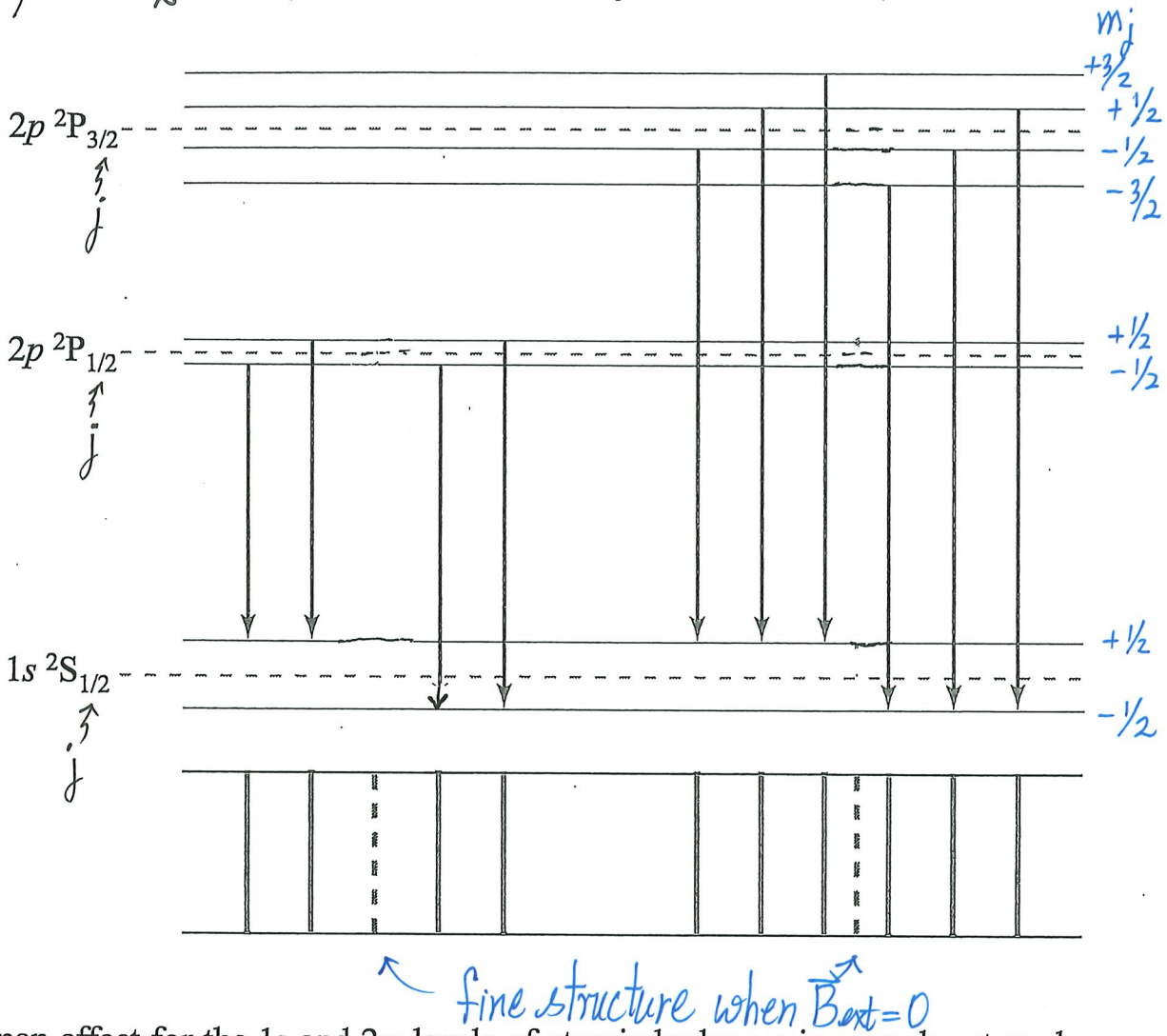
spectrum



due to spin-orbit interaction (fine structure)

$\left(\hat{H}_{atom} + \hat{H}'_{so}\right) + \hat{H}'_z$ ($\vec{B}_{ext} \neq 0$, but weaker than $\vec{B}_{int}$ in spin-orbit interaction)

Hydrogen
($n=2 \rightarrow n=1$)



Selection rules for allowed transitions

$\Delta m_j = 0, \pm 1$

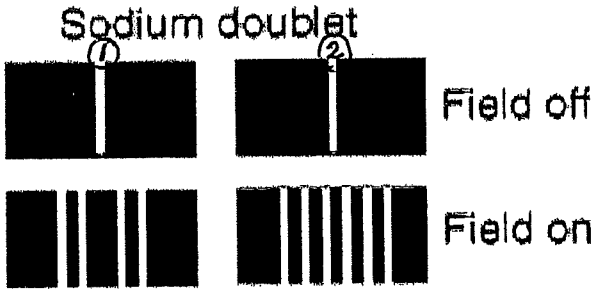
10 lines in weak $\vec{B}_{ext}$

Anomalous Zeeman Effect

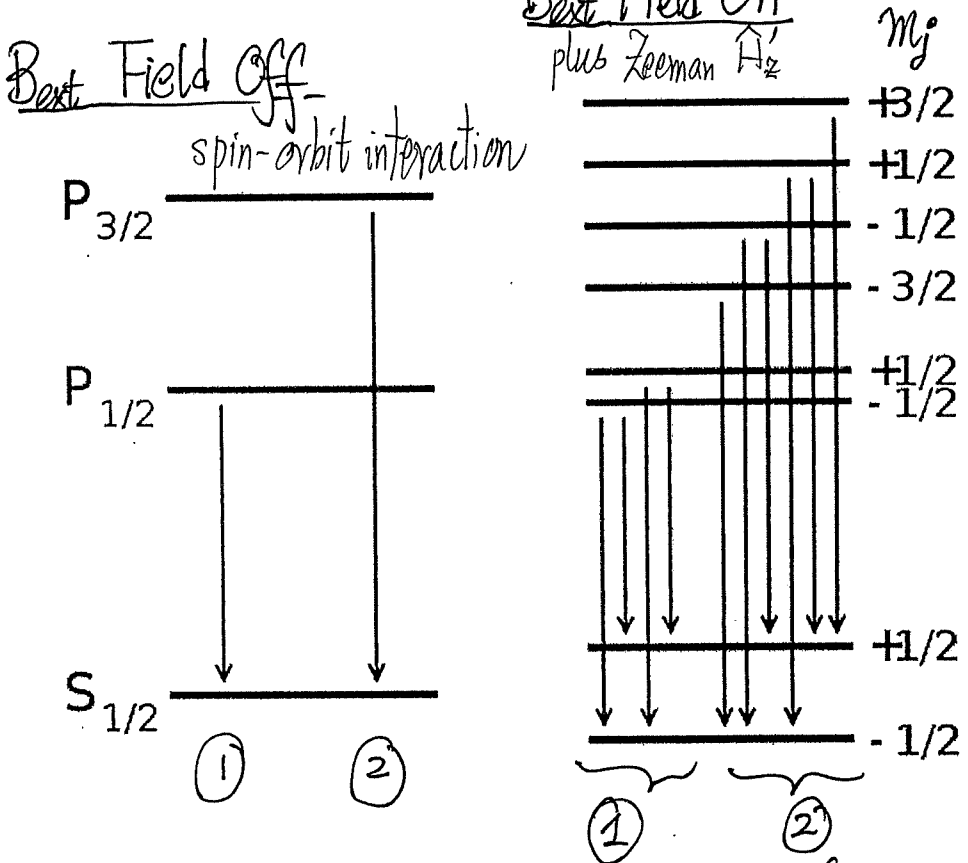
The Zeeman effect for the 1*s* and 2*p* levels of atomic hydrogen in a weak external magnetic field, showing the allowed transitions. A schematic diagram of the resulting spectrum is shown at the bottom. The dashed lines show the fine structure that is present in the absence of an external magnetic field.

Same Works for Sodium

- $\hat{H}'_{so}$ only
  Sodium D lines (doublet)
- With $\vec{B}_{ext}$, each line in doublet split
- This example uses Na atom
- Bigger atoms have stronger $\hat{H}'_{so}$
- Thus, weak $\vec{B}_{ext}$ case is often observed in heavier atoms



Zeeman term leads to $g_{Lande} \mu_B m_j B_{ext}$



$\Delta m_j = 0, \pm 1$ selection rules

Final Remarks

- The study on Zeeman effect had led to key developments in ideas/concepts in Quantum Mechanics
- QM and spin, plus techniques (approximations), are key to understand Fine structure and Zeeman effect
- Ideas: $\vec{J}$, spin-orbit interaction $\hat{H}'_{so}$, $\hat{H}_z$, strong/weak field cases...
  are introduced within the context of hydrogen atom here.
  But ideas/techniques are applicable to other atoms
  [need to add up $\vec{L}$'s and $\vec{S}$'s of many electrons]
- Historically, $g_{\text{Landé}}$ was introduced (1921) to describe exp'tal data before QM and spin were established. Its value informs us how important spin-orbit interaction is in an atom.

An appreciation...

Modern/Quantum Physics level : Descriptive
- Zeeman : "$-\vec{\mu}_L \cdot \vec{B}$"
- Spin-orbit: "$\vec{S} \cdot \vec{L}$"
- Hydrogen's relative correction: "ignored"
- Strong & Weak Field Zeeman: "not treated"

Applied Quantum Mechanics:

A bit of Quantum Mechanics (TISE, Angular Momenta in QM, Think classical and go quantum (look for $\hat{H}'_{so}$ and $\hat{H}'_z$)) and approximation methods (perturbation theory, physical sense) can take us to a deeper and quantitative understanding that backs up the qualitative descriptions.